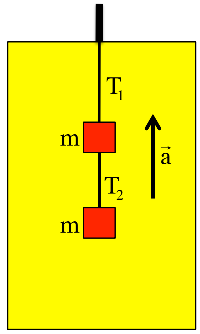
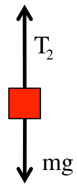
Problem 5.31



a.) Determine the tensions:

This is one of those situations in which it is not, maybe, really easy to see what's happening from a Quick and Dirty perspective, what with the internal tensions acting as they do. Using the Formal approach is the way we'll go. As such:

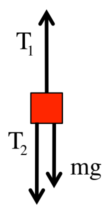
lower mass:

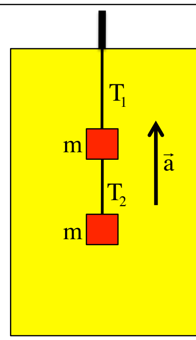


so . . .

$$\sum F_y :$$
$$\Rightarrow \quad T_2 - mg = ma$$
$$\Rightarrow \quad T_2 = mg + ma$$
$$= (3.50 \text{ kg})(9.80 \text{ m/s}^2) + (3.50 \text{ kg})(1.60 \text{ m/s}^2)$$
$$= 39.9 \text{ N}$$

upper mass:

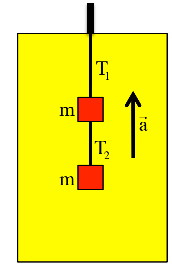




so . . .

$$\sum F_y :$$
$$\Rightarrow \quad T_1 - T_2 - mg = ma$$
$$\Rightarrow \quad T_1 = mg + ma + T_2$$
$$= (3.50 \text{ kg})(9.80 \text{ m/s}^2) + (3.50 \text{ kg})(1.60 \text{ m/s}^2) + (39.9 \text{ N})$$
$$= 79.8 \text{ N}$$

$$\sum F_y :$$
$$\Rightarrow \quad T_1 - mg - mg = 2ma$$
$$\Rightarrow \quad T_1 = 2m(g + a)$$
$$= 2(3.50 \text{ kg})[(9.80 \text{ m/s}^2) + (1.60 \text{ m/s}^2)]$$
$$= 79.8 \text{ N}$$



Now that we have the solutions, and just for the amusement of it, let's try the Quick and Dirty approach.

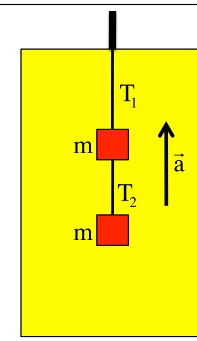
The forces affecting the total mass come from the top tension and gravity on both bodies. Using the Q&D approach, then:

$$\sum F_1 :$$
$$\Rightarrow \quad T_1 - mg - mg = (2m)a$$
$$\Rightarrow \quad T_1 = 2m(g + a)$$
$$= 2(3.50 \text{ kg})[(9.80 \text{ m/s}^2) + (1.60 \text{ m/s}^2)]$$
$$= 79.8 \text{ N}$$

For the bottom mass, it's the middle tension and the gravity on the bottom body, or:

$$\sum F_2 :$$
$$\Rightarrow \quad T_2 - mg = ma$$
$$\Rightarrow \quad T_2 = m(g + a)$$
$$= (3.50 \text{ kg})[(9.80 \text{ m/s}^2) + (1.60 \text{ m/s}^2)]$$
$$= 39.9 \text{ N}$$

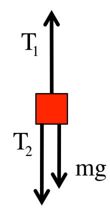
Same solutions: It worked!



b.) If the maximum tension allowable for the string is 85 N, what is the maximum acceleration the system can handle?

The string that is going to have the most to deal with will be the top string as it will have to support both masses. Rewriting its N.S.L. expression, we get:

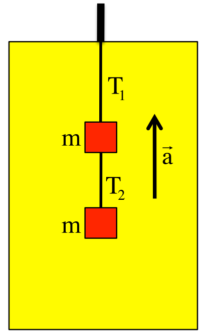
upper mass:



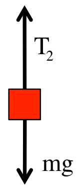
so . . .

$$\sum F_y :$$
$$\Rightarrow \quad T_1 - T_2 - mg = ma$$
$$\Rightarrow \quad a = \frac{T_1 - T_2 - mg}{m}$$

To solve this, we need the lower tension. To that end:



lower mass:



so . . .

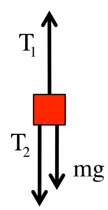
$$\sum F_y :$$

$$\Rightarrow \quad T_2 - mg = ma$$

$$\Rightarrow \quad T_2 = mg + ma$$

Substituting into our original "upper mass" expression, we get:

upper mass:



$$\sum F_y :$$

$$\Rightarrow \quad T_1 - T_2 - mg = ma$$

$$\Rightarrow \quad a = \frac{T_1 \quad - \quad T_2 \quad - mg}{m}$$

$$\Rightarrow \quad a = \frac{(85.0\text{ N})}{m} - \frac{[mg + ma]}{m} - \frac{mg}{m}$$

$$\Rightarrow \quad a = \frac{(85.0\text{ N})}{m} - g - a - g$$

$$\Rightarrow \quad 2a = \frac{(85.0\text{ N})}{m} - 2g$$

$$\Rightarrow \quad a = \frac{(85.0\text{ N})}{2m} - g$$

$$\Rightarrow \quad a = \frac{(85.0\text{ N})}{2(3.50\text{ kg})} - (9.80\text{ m/s}^2)$$

$$= 2.34\text{ m/s}^2$$